Math 2002 Midterm

No calculators or other aids are allowed. Be sure to write your name and student ID# on the booklet provided. The first question is worth 4 points, and the rest are each worth 6. You have two hours. Good luck!

1. (a) State the definition of the double integral $\int_R f(x,y)\,dA$, where R is the rectangle $[a,b] \times [c,d]$.
   (b) State Green's Theorem.

2. Estimate the value of the integral
$$\int_D ye^{x^2}\,dA$$
using $n=3$, $m=3$, lower left endpoints, and D is the region bounded by the curves $y=x$, $y=0$, and $x=3$.

3. Find the volume of the solid bounded by the cylinder $y^2+z^2=4$ and the planes $x=0$, $x+z=3$.

4. For each of the following two vector fields $\mathbf{F}$, determine if it is conservative, and if it is, find a function f such that $\bigtriangledown f = \mathbf{F}$.
   (a) $\mathbf{F} = (2e^{2x} - y\sin xy)\mathbf{i} + (-x\sin xy + e^{2y})\mathbf{j}$.
   (b) $\mathbf{F} = (e^x \ln y + x)\mathbf{i} + (e^{\frac{x}{y}})\mathbf{j}$.

5. Evaluate the integral $\int_C x^2y\,ds$, where C travels around the upper-half of the circle $x^2+y^2=9$ in the counter-clockwise direction.

6. Find the value of the integral $\int_C \mathbf{F}\cdot dr$, where C is the line from $(0,0)$ to $(1,2)$, followed by the line from $(1,2)$ to $(3,0)$, followed by the line from $(3,0)$ to $(0,0)$, and $\mathbf{F} = (y^2+e^x)\mathbf{i} + (\cos(y^2)+4xy)\mathbf{j}$.

7. Suppose an ant begins at the point $(0,4)$ in the plane. It then walks in a path around the plane, with its position after t hours given by the equation
$$r(t) = (t^3, t^2+4)$$
As it walks, wind swirls around the plane; its vector field is given by $\mathbf{F} = (-y)\mathbf{i} + (x)\mathbf{j}$. If the ant walks for one hour, how much work is the wind doing on the ant? Is the wind helping or hindering the ant's progress?

8. (Bonus question, +5 marks) Prove the fundamental theorem for line integrals.

Spherical co-ordinates:

$$x = r\sin\phi\cos\theta,\ y = r\sin\phi\sin\theta,\ z = r\cos\phi$$

the Jacobian for the spherical co-ordinates transformation is $r^2\sin\phi$.