Math 1000 Summer Section 02 Midterm

Instructions:

- Write your name and student ID number on the front of the booklet.

- Show all work, and circle your final answer.

- No calculators, notebooks, or other aids are allowed.

- Each question is worth 4 marks, and there are 9 questions, so the exam is out of 36.

- You have 1 1/2 hours.

Good luck!

1. (a) State (in words) what

$$\lim_{x \to a} f(x) = L$$

   means.

   (b) Define the derivative of $f(x)$ at a. (Either form is acceptable).

2. Determine, for the following function f(x):

   (a) Where $f'(x)$ does not exist.

   (b) Where the tangent line to $f(x)$ is horizontal.

3. For the following function:

$$f(x) = \begin{cases} 4 - x^3 & \text{if } x < 1; \\ x^2 + a^2 & \text{if } x \geq 1. \end{cases}$$

at what value(s) of a is $f(x)$ continuous at 1?

4. Find the value of the following limit:

$$\lim_{x \to -1} \frac{\sqrt{x + 5} - 2}{x^2 - 1}$$

5. Find the derivative of the following function *using the definition of the derivative*, and not any rules of differentiation: $f(x) = \dfrac{1}{x^2}$ at $a = 3$.

6. Find the derivative of the following functions; you may use rules of differentiation for these questions.

   (a) $f(x) = (x^3 + 9x^2)^{15}$

   (b) $f(x) = \dfrac{\tan x}{\sec x}$

7. Find the equation of the tangent line to the curve $y^4 + \sin x = x^2 + 16$ at the point $(0, 2)$.

8. A bumblebee moves around in a strange pattern - its distance from the hive after t minutes is given by the function $\ln(\sin t)$. At what time(s) is the bumblebee not moving between 0 and π minutes?

9. Meghan and Andrew meet at a restaurant. At 1pm, Andrew leaves the restaurant and walks east at a rate of 2km/h. At 2pm, Meghan leaves the restaurant and begins walking north at a rate of 4 km/h. How fast is the distance between them increasing at 4pm?

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