

## CPSC 513 Winter 2010 Midterm

No outside aids (calculators, textbooks, etc.) are allowed. Good luck!

1. (a) Define what it means for a function to be computable.  
(b) Show that the function

$$f(x) = \text{the largest } n \text{ such that } n^3 \leq x$$

is computable.

2. (a) Define what it means for a partial function to be  $\mu$ -recursive.  
(b) Show that every partially computable function is  $\mu$ -recursive.
3. Suppose a sequence  $f(n)$  is given by  $f(0) = 2, f(1) = 3, f(2) = 7$ , and for  $n > 2$ ,

$$f(n) = f(n-1) + f(n-2) + f(n-3).$$

Show that  $f(n)$  is primitive recursive.

4. Give the statements of:
  - (a) the Universality Theorem,
  - (b) the Parameter Theorem.
5. Let  $A$  and  $B$  be recursively enumerable sets. Show that the set

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

is recursively enumerable.

6. (a) State Rice's Theorem.  
(b) Let  $W_x = \{n : \Phi(n, x) \downarrow\}$ . Show that the sets

$$\{x : W_x \text{ is infinite}\} \text{ and } \{x : W_x \text{ is finite}\}$$

are not recursive.