CPSC 513 Midterm Information

The midterm is next Friday, March 5th, 9:00-9:50. No outside aids (calulators, textbooks, etc.) are allowed.

You will be expected to know the following definitions:

- computable and partially computable function;
- primitive recursive, PRC class, μ-recursive;
- recursive set, recursively enumerable set;
- many-one reducible, m-complete;

as well as the statement of the following theorems:

- universal theorem, normal form theorem;
- parameter theorem;
- Rice's theorem;

and how one can use those theorems. You will not need to know how to reconstruct a program from its number, nor how to find the number of a given program.

Here are some sample questions. (note: this is more questions than you will get on the midterm itself!)

1. Write a program in the language P that adds X_1 and X_2 (without using macros).
2. Let $P(x)$ be a computable predicate. Show that

   $$f(x) = \begin{cases} 1 & \text{if there are at least } x \text{ numbers } n \text{ such that } P(n) = 1; \\ \uparrow & \text{else.} \end{cases}$$

   is partially computable.
3. (a) State the definition of a primitive recursive function.

   (b) Show that any primitive recursive function is computable.
4. Let $S(x)$ be true if x is the sum of two perfect squares, false otherwise. Show that $S(x)$ is primitive recursive.
5. (a) State what it means for a class of functions to be a PRC class.

   (b) Show that if the function $f(t, x_1, \cdots x_n)$ is in a PRC class **C**, then so is the function

   $$g(y, x_1, \cdots x_n) = \sum_{t=0}^{y} f(t, x_1 \cdots x_n)$$

6. (a) State the definitions of recursive set and recursively enumerable set.
   (b) Show that the set $K = \{n : \Phi(n, n) \downarrow\}$ is not recursive but is recursively enumerable.
7. Show that if A and $\bar{A}$ are recursively enumerable, then A is recursive.
8. (a) State the parameter theorem.
   (b) Let $f(x, y)$ be a partially computable function. Show that there is a primitive recursive function $g(u, v)$ such that

$$\Phi(x, g(u, v)) = f(\Phi(x, u), \Phi(x, v))$$

9. (a) Say what it means for $A \leq_m B$.
   (b) Show that for any A, $A \leq_m A$.
   (c) Show that if $A \leq_m B$ then $\bar{A} \leq_m \bar{B}$.
10. Show that $\bar{K} \leq_m$ EMPTY.
11. (a) Say what it means for a set A to be m-complete.
    (b) Let $K_0 = \{n : \Phi(l(n), r(n)) \downarrow\}$. Show that K_0 is m-complete.
12. (a) State Rice's theorem.
    (b) Use Rice's theorem to show that the sets

$$\{n : \Phi(x, n) = x^2\} \text{ and } \{n : \{x : \Phi(x, n) \downarrow \text{ is finite}\}\}$$

are not recursive.