CPSC 513 Final Exam Information

The final exam is next Friday, April 23rd, 12:00-3:00, in MS 217. No outside aids (calulators, textbooks, etc.) are allowed.

You will be expected to know the following definitions:

- computable and partially computable function;
- primitive recursive, PRC class, μ-recursive;
- recursive set, recursively enumerable set;
- many-one reducible, m-complete, m-closed;
- one-one reducible, 1-complete, 1-closed;
- Turing reducible, T-complete, T-closed;
- for a total function G, G-partially computable, G-recursive, and G-recursively enumerable;
- the jump G' of a total function G;
- Arithmetic Hierarchy definitions $(\Sigma_n, \Pi_n, \Delta_n)$;

as well as the statement of the following theorems:

- normal form theorem;
- universal theorem and the parameter theorem (and their relatived versions);
- Rice's theorem and the Rice-Shapiro theorem;
- Recursion theorem;
- Kleene's Hierarchy theorem;

and how one can use those theorems. You will not need to know how to reconstruct a program from its number, nor how to find the number of a given program.

Here are some sample questions.

1. (a) State the definition of a primitive recursive function.
   (b) Show that any primitive recursive function is computable.
2. Let $\gcd(x, y)$ be the greatest common divisor of x and y. Show that $\gcd(x, y)$ is primitive recursive.
3. (a) State what it means for a class of functions to be a PRC class.

(b) Show that if the function $f(t, x_1, \cdots x_n)$ is in a PRC class $\mathbf{C}$, then so is the function

$$g(y, x_1, \cdots x_n) = \prod_{t=0}^{y} f(t, x_1 \cdots x_n)$$

4. (a) State Rice's theorem.

   (b) Use Rice's theorem to show that the sets

$$\{n : \Phi(5, n) = 16\} \text{ and TOT}$$

   are not recursive.

5. (a) State the Rice-Shapiro theorem.

   (b) Show that INF is not RE.

   (c) Show that EMPTY is not RE.

   (d) Is the set $\{n : \Phi(5, n) = 16\}$ RE?

6. Show that there exists partially computable functions f, g so that

$$f(0) = 3$$

$$g(0) = 5$$

$$f(2t) = g(t) + 1$$

$$g(t + 1) = f(2t + 2)$$

7. Write a program in the language P_m that strictly computes the function

$$s(u, v) = \begin{cases} s_1 & \text{if } u \text{ is a substring of } v; \\ 0 & \text{else.} \end{cases}$$

   on the alphabet $\{s_1, \ldots, s_m\}$.

8. Write a Post-Turing program that takes a string $w \in \{s_1, s_2\}^*$ and returns a string in $\{s_1, s_2\}^*$ whose number is one less than that of w's (if the string is empty, return empty).

9. Give a Turing Machine that returns the last symbol of a string $w \in \{s_1, \ldots, s_m\}^*$.

10. (a) State the relativized universality theorem.

    (b) State the relativized and strengthened parameter theorem.

11. (a) If G is a total function, define the jump of G, G'.

    (b) Show that G' is G-RE but not G-recursive.

12. (a) Define Σ_n.

(b) Show that for each $n \geq 0$, $\emptyset^{(n)}$ is in Σ_n.

13. Show that if A and $\bar{A}$ are both G-RE, then A is G-recursive.

14. Show that if $A \leq_T B$, then $A' \leq_1 B'$.

15. (a) Define what $A \leq_1 B$ means.

    (b) Show that $K \leq_1$ EMPTY.

16. (a) Define what $A \leq_T B$ means.

    (b) Show that if $A \leq_m B$, then $A \leq_T B$.