

Assignment 6: Reducibility and Oracles

This assignment is due Friday April 16th, at the beginning of class (9:00am).

1. Recall that the strengthened Parameter theorem says that for $n, m > 0$,

$$S_m^n(u_1, \dots, u_n, y) = S_m^n(u'_1, \dots, u'_n, y)$$

implies that $u_1 = u'_1, \dots, u_n = u'_n$.

- (a) Show, by giving a counter-example, that the statement for any $n, m > 0$,

$$S_m^n(u_1, \dots, u_n, y) = S_m^n(u'_1, \dots, u'_n, y')$$

implies that $u_1 = u'_1, \dots, u_n = u'_n, y = y'$ is *not* true.

- (b) Under what conditions on y and y' is it true?

2. Prove that $K \leq_1 \text{FIN}$.
3. For any sets $D, E \subseteq \mathcal{N}$, define

$$D \oplus E := \{2x : x \in D\} \cup \{2x + 1 : x \in E\}$$

Now, suppose that $A \subseteq \mathcal{N}$, with $K \leq_t A$, and define

$$C = \{x \in K : \Phi(x, x) \notin A \oplus \bar{A}\}.$$

Prove that:

- (a) $C \leq_t A$,
 - (b) $A \leq_1 C$.
4. For any total function g , show that if B and C are g -R.E., then so are $B \cup C$ and $B \cap C$.
 5. For sets A, B, C does A being B -R.E., and B being C -R.E. imply A is C -R.E.? Either prove or give a counter-example.