Assignment 5: Recursion Theorem, Strings, Turing Machines

This assignment is due Monday, April 5th, at the beginning of class (9:00am).

1. (a) Use the Recusion theorem to show that there is a partially computable function f that satisfies the equations

$$f(x,0) = x+2$$

$$f(x,1) = 2f(x,2x)$$

$$f(x,2t+2) = 3f(x,2t)$$

$$f(x,2t+3) = 4f(x,2t+1)$$

   (b) Show that f is total.

2. Determine the following:

   (a) for the alphabet $\{s_1, s_2\}$, the number of the string: $s_1s_2s_1s_2$;

   (b) for the alphabet $\{s_1, s_2, s_3\}$, the number of the string: $s_3s_2s_3s_1$;

   (c) for the alphabet $\{s_1, s_2\}$, the string with number 100;

   (d) for the alphabet $\{s_1, s_2, s_3, s_4\}$, the string with number 100.

3. Let f be a function $\{s_1, \cdots, s_n\}^* \to \{s_1, \cdots s_n\}^*$ which returns s_1 if a string w has a even number of symbols, and 0 otherwise. Write a program in P_n that computes the function f.

4. Write a Post-Turing program using that strictly computes the function $s(x) = x+1$ relative to $\{s_1, s_2\}$ (that is, its input is a string $w \in \{s_1 \dots s_2\}$, and its output is the string in $\{s_1, s_2\}$ corresponding to the number of w + 1).

5. If $u \neq 0$, let $n(u,v)$ be the number of occurrences of u as a part of v (for example, $u(ba, ababaa) = 2$); and let $u(0,v) = 0$. Give a Turing machine that strictly computes n.