Assignment 4: Parameter Theorem, R.E. Sets, Reducibility, Rice-Shapiro Theorem

This assignment is due Friday, March 19th, at the beginning of class (9:00am).

1. Suppose that $f : \mathcal{N} \to \mathcal{N}$ is a strictly increasing function: in other words, $f(n+1) > f(n)$ for all $n \in \mathcal{N}$. Prove that the set

   $$B = \{f(n) : n \in \mathcal{N}\}$$

   is recursive.

2. Show that every infinite recursively enumerable set B has an infinite subset $B' \subseteq B$ which is recursive (hint: use the previous question).

3. Suppose A and B are subsets of $\mathcal{N}$. Prove the following properties of many-one reducibility:

   (a) $A \leq_m B$ if and only if $\bar{A} \leq_m \bar{B}$;

   (b) if A and B are m-complete, then $A \equiv_m B$;

   (c) if A is m-complete, then A is not recursive.

4. Let INF $= \{x \in \mathcal{N} : W_x \text{ is infinite}\}$. Show that TOT $\equiv_m$ INF.

5. Show that each of the following sets are not recursively enumerable by using the Rice-Shapiro Theorem:

   (a) INF $= \{x \in \mathcal{N} : W_x \text{ is infinite}\}$;

   (b) FIN $= \{x \in \mathcal{N} : W_x \text{ is finite}\}$;

   (c) PREDICATE $= \{x \in \mathcal{N} : \Phi_x \text{ is a predicate}\}$.