Assignment 3: Numbering and Universal Programs

This assignment is due Friday, February 26th, at the beginning of class (9:00am).

1. The Fibonacci sequence is given by

   - $F(0) = 0$,
   - $F(1) = 1$,
   - $F(n+2) = F(n+1) + F(n)$.

   Show that F is primitive recursive. (Hint: use the pairing function).

2. A function f is given by "unnested double recursion" if there are functions g_1, g_2, h such that

   - $f(0, y) = g_1(y)$,
   - $f(x+1, 0) = g_2(x)$,
   - $f(x+1, y+1) = h(x, y, f(x, y+1), f(x+1, y))$.

   Show that if g_1, g_2, and h are all in some PRC class **C**, then so is f. (Hint: use the functions $[a_1, \cdots a_n]$).

3. Suppose the number of the program P is $(2^{46})(3^0)(5^2)(7^{37}) - 1$. Write out the code for P, and determine what it returns if given the input X_1.

4. Suppose we define a predicate $H(x)$, which is true exactly when the program with number $r(x)$ halts on input $l(x)$. Show that H is not computable.

5. Suppose that $f(x_1, \cdots x_n)$ is computable by some program P. Suppose we also know that there is some primitive recursive function $g(x_1, \cdots x_n)$ such that

   $$\mathrm{STP}^{(n)}(x_1, \cdots x_n, \#(P), g(x_1, \cdots x_n))$$

   is always true. Show that f is primitive recursive. (In other words, if the amount of time the program takes to run is bounded by some primitive recursive function, then the original function is primitive recursive).