

## Assignment 1: Proofs and Simple Programs

This assignment is due Friday, January 22nd, at the beginning of class (9:00am).

1. If  $\mathcal{E}$  is the set of even numbers (0, 2, 4, 6...), and  $\mathcal{N}$  the set of natural numbers (0, 1, 2, 3...), give examples of:

- (a) A 1-1 function from  $\mathcal{N}$  to  $\mathcal{E}$ ,
- (b) An onto function from  $\mathcal{E}$  to  $\mathcal{N}$ .

Be sure to *prove* that the first function is 1-1, and the second function is onto!

2. Prove that the composite of two isomorphisms is an isomorphism.
3. Use strong induction to prove that any amount of postage of 12 cents or more can be achieved by using some combination of 4 and 5 cent stamps.
4. Write a program in the language  $\mathcal{P}$  that returns 1 if the input is odd, and 0 if the input is even (without using macros).
5. Write a program in the language  $\mathcal{P}$  that computes the function  $f(x) =$  the greatest natural number  $n$  such that  $n^2 \leq x$ . You may use any macros we have discussed in class.