CPSC 413 Fall 2010: Midterm

November 2nd, 2010

Name: ________________________________

ID#: ________________________________

Instructions:

- No outside aids of any kind are allowed.
- You have 1 hour and 15 minutes.
- Do not turn this page until you are told to do so.
- There are four questions on this exam, and each question is worth a total of 10 points.

1. (a) Define what it means for $f = O(g)$ (3 pts).

   (b) Show that $ln(n^4) = O((ln(n))^2)$ (3 pts).

   (c) Suppose that we have two non-negative functions f, g such that for $n \geq 100$, $f(n) \leq g(n)$. Show that $f + g = O(g)$ (4 pts).

2. Meghan is planning a cross-country trip along the Trans-Canada highway from Halifax to Vancouver. She knows the location of n gas stations along the trip (not neccesarily in order). Her car can only run k kilometers before she must stop for gas.

   (a) Give an algorithm that can determine the least number of gas stations she must stop at along her journey so that she never runs out of gas, and show that your algorithm runs in $O(n \log n)$ time (4 pts).

   (b) Prove that your algorithm always return the minimum number of gas stations (6 pts).

3. Suppose that a divide and conquer algorithm splits an input of size n into 3 pieces, each of size $\frac{n}{3}$, while the time taken to split and recombine the data is $O(n)$. That is, if $T(n)$ is the running time of the algorithm on n inputs, then there is some constant c so that

$$T(n) \leq 3T(n/3) + cn \text{ and } T(3) \leq c.$$

Show that the running time of the algorithm is $O(n \log n)$ (10 pts).

4. Given an array $[a_1, a_2, \ldots a_n]$ of distinct integers, say that a_i is a *local minimum* if a_i is strictly less than both of its neighbours (an endpoint is considered a local minimum if it is strictly less than its single neighbour).

   (a) Give an algorithm for finding a local minimum while only looking at $O(\log n)$ elements of the array (5 pts).

   (b) Prove that your algorithm does find a local minimum (5 pts).