CPSC 413: Exercise Set 9

1. As a freelance writer, each week, you can choose between two different jobs. The jobs change from week to week, but each week, one of the jobs will require travel, and the other will not. If you choose the job that requires travelling, then you will need to recover from the travel, so that you cannot work the following week. Suppose you know ahead of time the different jobs for the next n weeks. If you take the non-travel job in week i, you get l_i dollars. If you choose the travel job in week i, you get m_i dollars, but, as mentioned above, you cannot work the following week. Obviously, you would like to make the most amount of money. For example, suppose the amounts for the jobs each week were

   $$l_1 = 300, l_2 = 200, l_3 = 250$$

   and

   $$m_1 = 400, m_2 = 600, m_3 = 300$$

   then the choice which makes the most money would be to to take the non-travel job in the first week (giving \$300) then take the travel job in the second week (which takes up both weeks 2 and 3), giving \$600, for a total profit of \$900. (Note that you can choose to take the travel job in the 3rd week, but in this case, this does not give an optimal solution). Devise a dynamic programming solution to the problem to help decide which jobs to take each week.

2. Recall from the midterm review class the following problem: a company is reviewing the performance of a stock over the past n days. Say that the stock has value v_i on day i. The problem is to determine the optimal point the company should have first bought, then later sold, the stock to make the most profit. For example, if the stock values were

   $$5, 2, 3, 7, 8, 1$$

   then the best profit would be to buy on day 2 for \$2, then sell on day five for \$8, for a profit of \$6. In class, we showed that one could use divide and conquer to give an optimal solution in $O(n \log n)$ time. Use dynamic programming to give a solution that runs in $O(n)$ time.