CPSC 413: Exercise Set 5

1. A company is selling off some of its old stock, one item per month. Each of the *n* items is currently worth \$500, but as each month passes, its value depreciates by some factor $r_i < 1$, so that after *t* months, the company call only sell it for $500 \cdot r_j^t$ (for example, if $r_i = 0.5$, then after two months, item *i* will only sell for $500 \cdot (0.5)^2 = \$125$). Which order should the company sell their items off so as to maximize their profit? Find a greedy algorithm that gives an optimal solution in $O(n \log n)$ time.

Update! Unfortunately, there is no natural greedy algorithm for this problem. There are only two choices for a greedy algorithm: sell the items in increasing r_i value, or in decreasing r_i value. If we have $r_1 = \frac{1}{2}$, $r_2 = \frac{3}{4}$, and $r_3 = \frac{1}{100}$, then the optimal solution is in the order given, which is in neither increasing or decreasing order of r_i . My apologies!

2. Consider a complete binary tree T with n nodes (complete means every non-leaf node has two children) Suppose that each node v is labelled with some distinct real number f(v). Say that a node v is a local minimum if f(v) is less than the f value of each node connected to v. Give a divide and conquer algorithm that finds a local minimum of the tree while only looking at $O(\log n)$ nodes.