# CPSC 413: Exercise Set 4

1. Suppose that G is an undirected graph. Prove that if any two of the following statments are true, then so is the third:

   - $G$ is connected;
   - $G$ has no cycles;
   - $G$ has one less edge than the number of vertices.

   (So, one can take any two of these items as the definition of a tree).

2. Here is another algorithm that tries to find a minimum spanning tree of a graph: start with an empty set of edges $T$. Go through each edge $e$ of the graph in some order, and add the edge $e$ to $T$. If adding this edge causes a cycle to appear in $T$, then remove the highest-cost edge in that cycle. Repeat until all edges have been considered. Does this give a minimum spanning tree? Why or why not?

3. Suppose we have a minimum spanning tree $T$ of a graph $G$, and add an edge between two existing vertices in the graph. Is it possible to determine in $O(|E|)$ time whether $T$ is still a minimum spanning tree?