CPSC 413: Exercise Set 12

1. It is nearing the end of the semester, and you are trying to decide on which courses to spend your time studying. For each of the n courses you are taking, you have made a function which estimates what grade you think you will get if you spend t hours studying on course i: call this quantity Grade(i, t) (assume all grades are out of 4.0).

   Of course, you only have a limited number of hours to study - say you have H total hours available to study. You need to decide how many hours to spend on each course so that your estimated average grade is as high as possible (assume that the number of hours you spend per course is always an exact integer number of hours). Design a dynamic programming solution to this problem that is polynomial in the variables n and H.

2. We have seen one problem about clauses: given a set of clauses, the problem **SAT** asks whether there is an assignment of terms to bits so that all clauses are satisfied. Here is another problem relating to clauses. Call a clause C *monotone* if each term in the claus is a variable (not its negation). So, for example, $(x_1 \vee x_2)$ is monotone, while $(x_1 \vee \overline{x_2} \vee x_3)$ is not. Obviously, any set of monotone clauses is satisfiable: set all variables to 1. So, the monotone satisfiability problem asks a different question: given a set of clauses over variables $x_1, x_2, \ldots x_n$, is there a set of k or fewer variables to which we assign 1 (and all the rest 0) so that the clause is still satisfiable? For example, the clauses

   $$(x_1 \vee x_2), (x_2 \vee x_3), (x_3 \vee x_4 \vee x_5)$$

   are all satisfiable if we set $x_2 = 1, x_4 = 1$, but there is no way to only assign one variable 1 and satisfy all clauses. Show that this problem is NP-complete (hint: reduce the vertex cover problem to it).